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The E_{1+}/M_{1+} and S_{1+}/M_{1+} ratios of $\gamma N \to \Delta(1232)$ with a point-form relativistic quantum mechanics

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Abstract

The point-form relativistic quantum mechanics is employed to study the photo- and electro-productions of the nucleon resonances. Both the ratios of E_{1+}/M_{1+} and S_{1+}/M_{1+} of $\gamma N \to \Delta$ transition are calculated. Configuration mixing effect is simply included. The results of the point-form relativistic quantum mechanics indicate that the relativistic effects provide a remarkable role both on the transition amplitudes and on the two ratios E_{1+}/M_{1+} and S_{1+}/M_{1+}. It is found that small deformations of the nucleon and Δ resonance wave functions in D-wave can provides more sizable ratios in the point-form relativistic quantum mechanics than in the conventional non-relativistic constituent quark model.

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1. Introduction

It is known that the non-relativistic constituent quark model (NRCQM) has been employed to study low-energy hadronic phenomena for a long time. The discussion of the photo- and electro-production amplitudes of low-lying nucleon resonances with this approach is one of the most interesting topics since it closely relates to the non-perturbative QCD and it provides detailed messages about the complex nature of the nucleon. It is believed that the new data of nucleon structure functions in the resonance region and of the measurement of the nucleon resonance transition amplitudes can tell from different model calculations. In the past, most of studies were based on conventional NRCQM where the relativistic corrections both to the electromagnetic transition operators and to the nucleon and its resonance wave functions [1–3] were included. Since the conventional long wavelength approximation in the reduction of the electromagnetic transition operators of the NRCQM (see Ref. [2]) is not a good one if the virtuality of the photon Q^2 is large, say 0.5 GeV^2, and since the heavier the resonance is, the smaller valid range of the long wavelength approximation becomes, one concludes that the calculated transition amplitudes in the NRCQM are expected to be only valid in the region of very small Q^2.

Dirac first proposed three equivalent forms of relativistic dynamics [4]. They are instant-form, light-front form, and point-form (PF). We know that the instant and light-front forms are two rather popular approaches in the past several decades, however, the point-form is less known. In fact, the point-form relativistic framework has been discussed in detail by Keister and Polyzou [5] in 1991 and recently been carefully and systematically studied by Klink [6], and by many others [7–9]. It was also employed in the calculations of the nucleon form factors [7], of nucleon resonance strong decays [10], of the photo- and electro-productions of the nucleon resonances [11], and of some other aspects of hadron physics [12–14]. The results of the point-form constituent quark model (PFCQM) in the literature show the importance of the relativistic description. Particularly, when the momentum transfer Q^2 is at a moderate region of > 0.5 GeV^2,

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the differences between the NRCQM calculation and the fully relativistic PFCQM become evident. In the study of the proton electromagnetic and axial-vector form factors [7], it is found that the PF relativistic description always reduces the theoretical estimates of the NRCQM. The calculations of Ref. [7] and of the Q^2-dependences of the nucleon resonance electromagnetic transition amplitudes in the PFCQM [11], have already shown the advantages of the PFCQM and shown the remarkable differences from a NRCQM calculation. So far, how well the PF relativistic quantum mechanics in the understanding of hadron properties is still under investigation and discussion [8,14]. This is due to the problem of Poincaré space–time translation invariance in the PF relativistic quantum mechanics.

In this work, the point-form relativistic quantum mechanics will be employed to calculate the electromagnetic transitions of $\gamma N \to \Delta$, and particularly to study the ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} in the transition. Comparison to the results of the NRCQM will be displayed. It is believed that the relativistic effect will provide a remarkable influence on the two ratios. We expect that the magnitudes of the estimated ratios of the PFCQM are larger than those of the NRCQM, even if we use the same D-wave deformations from configuration mixing in the nucleon and the $\Delta(1232)$ resonance wave functions. In our previous work [11], the configuration mixing effect is missing in the calculation of $\Delta(1232)$ transition amplitudes. Thus, the obtained ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} are much less than the experimental values. It should be mentioned that the two ratios were also studied in the three forms of the relativistic quantum mechanics [15], where the qualitative D-wave deformations of the nucleon and $\Delta(1232)$ resonance are considered. Here, a configuration mixing effect, more realistic than the one taken in Ref. [15], is considered.

This Letter is organized as follows. In Section 2, the point-form relativistic quantum mechanics will be briefly discussed. Moreover, the electromagnetic transition amplitudes of the nucleon resonances will be displayed in this section. Numerical results and discussions will be displayed in Section 3. Finally, conclusions will be drawn in the last section.

2. Electromagnetic transition amplitudes in the point-form relativistic quantum mechanics

Here, we briefly review the framework of the point-form relativistic quantum mechanics. In the instant-form of the relativistic quantum mechanics, interactions are involved in P_0 (the time component of four-momentum) and in Lorentz boost operators J_{01}, J_{02}, and J_{03}. Therefore, the main difficulty of this framework is the lose of the manifest Lorentz covariance by construction because the three Lorentz boost operators contain the interactions. In the point-form, however, all the components of the four-momentum P_μ ($\mu = 0, 1, 2, 3$) are associated with the interactions. They are the Hamiltonians of the system. Other dynamical operators, like the angular momentum and Lorentz boost operators, are interaction free. Thus, the advantage of the PF is that all the Lorentz transformations remain purely kinematic and the theory is manifestly Lorentz covariant. In the PF relativistic quantum mechanics, one usually uses the Bakamjian–Thomas method (BJ) [16] by putting the interactions into a mass operator $\hat{M}$ to construct the interacting four-momentum operator P_μ. Since the BJ method and Dirac original work respectively imply hyperplane perpendicular to the velocity of the system [16,17] and hyperboloid surface, the BJ method, therefore, differs from the Dirac one. This difference has been recognized by Sokolov [18] and also discussed by Desplanques recently [19].

In the BJ method, the mass operator $\hat{M}$ can be divided into two parts. One is the free mass operator $\hat{M}_{\rm fr}$ without any interactions and the other one is the interacting mass operator $\hat{M}_{\rm int}$. The four-momentum P_μ relates to the mass operator by $P^\mu = \hat{M}\hat{V}^\mu_{\rm fr}$ with the free four-velocity operator $\hat{V}^\mu_{\rm fr}$ which is not affected by the interactions [17]. According to the commutation relations satisfied by the operators of the dynamical system and to the fact that P^μ is a Lorentz vector, one gets $[V^\mu_{\rm fr}, \hat{M}] = 0$ and $\hat{M}$ is a Lorentz scalar. Thus, the eigenstates of the four-momentum operator P^μ are the eigenstates of both the mass and the velocity operators. In the center-of-mass frame, the wave functions of the three-quark system can be obtained by solving a semi-relativistic Schrödinger equation (the non-relativistic kinetic energy operator is replaced by the semi-relativistic one including positive energy only). They are the eigenstates of the mass operator with interactions. Since in the PF, the Lorentz transformations remain purely kinematic, a so-called velocity state is usually introduced [6]

$$
\begin{aligned}
&|v; \vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3\rangle \\
&\quad = U_{B(v)}|k_1, k_2, k_3; \mu_1, \mu_2, \mu_3\rangle \\
&\quad = \prod_{i=1}^{3} D^{1/2}_{\sigma_i\mu_i}\big[R_W\big(k_i, B(v)\big)\big]|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle,
\end{aligned} \tag{1}
$$

where k_i ($i = 1$–3) is the quark momentums in the center-of-mass frame ($\sum_i \vec{k}_i = 0$). $B(v)$ is a Lorentz boost with four-velocity v. In Eq. (1) $p_i = B(v)k_i$, and $U_{B(v)}$ is a unitary representation of $B(v)$. $D^{1/2}(R_W)$ is the spin-1/2 representation matrix of the Wigner rotation $R_W(k_i, B(v)) = B^{-1}(B(v)k_i)B(v)B(k_i)$ [20]. A detailed discussion of the transformation properties of the velocity states has been given in Ref. [6]. It has been proved that all the Wigner rotations of a canonical boost of the velocity state are the same. Therefore, the spins can thus be coupled together to a total spin state as in the non-relativistic framework as well as in the center-of-mass frame. This is the practical advantage of using the velocity state in the PF relativistic mechanics.

To calculate the photo- and electro-production amplitudes of a nucleon resonance, we simply employ the PF spectator approximation in the electromagnetic interaction (see Refs. [6–15]). In the PF framework, the momentum transferred to the total nucleon is different from the momentum transferred to a struck constituent [19]. The conserved electromagnetic current operator contains both the one-body current and the dynamically determined current [21]

$$
J_\mu := j^1_\mu + j^{DD}_\mu. \tag{2}
$$

The one-body electromagnetic current j^1_μ in the point-form spectator approximation has the usual form of a point-like Dirac particle

$$\langle p'_i, \lambda'_i | j^1_\mu | p_i, \lambda_i \rangle = e_i \bar{u}(p'_i, \lambda_i) \gamma_\mu u(p_i, \lambda_i), \tag{3}$$

where $u(p_i, \lambda_i)$ is the Dirac spinor with momentum p_i and spin λ_i for the ith struck quark. The matrix element of the dynamically determined current is [21]

$$\langle p', \lambda'_i | j^{DD}_\mu | p, \lambda \rangle = -\frac{q_\mu + q^\perp_\mu}{q^2} q^\nu \langle p, \lambda'_i | j^1_\nu | p, \lambda \rangle, \tag{4}$$

where, q_μ is the four-momentum of the incoming photon and $q^\perp_\mu$ is a four-vector perpendicular to q and it is determined by the requirement that no pole at $q^2 = 0$. Clearly, the dynamically determined current affects the longitudinal current and it does not affect the transverse current. Moreover, the gauge invariant constraint condition $q^\mu J_\mu = 0$ is satisfied [21]. Here, it should be stressed that the construction of the dynamically determined current J^{DD} is not unique. There are other ways to construct J^{DD} [22] which is different from Eq. (4).

One can directly calculate the transition amplitudes by using the same approach as in the calculation of resonance strong decays in the PF [10]. We know that the transverse (longitudinal) transition amplitude is the matrix element of the transverse (longitudinal) polarized photon and quark interaction of H^T_{em} (H^L_{em}). In the NRCQM, the results are reference frame-dependent. It should be stressed that the results of the amplitudes in the fully relativistic PF description are frame-independent. Say H^T_{em} for example, the initial and final momenta $P_i = M_N V_i$ and $P_f := M_X V_f$, and the initial and final velocities V_i and V_f are chosen to be in z-direction. We have $U^\dagger(\Lambda) J_x(0) U(\Lambda) = J_x(0)$, where Λ is the Lorentz transformation from one frame and to another (for example the transformation from Breit frame to equal velocity reference frame [23]), and $U(\Lambda)$ is the unitary representation of the Lorentz transformation Λ with $U^\dagger(\Lambda)U(\Lambda) = 1$. Thus, for the transverse amplitudes, the predictions of the fully relativistic PF framework are reference frame-independent. Here, the amplitude is

$$\begin{aligned}
A_{\mu',\mu} &= \xi \langle f, \mu' | H^T_{\text{em}} | i, \mu \rangle = -\sqrt{\frac{2\pi\alpha_E}{\omega}} \xi \langle f, \mu' | \vec{J} \cdot \vec{\epsilon} | i, \mu \rangle \\
&= 3\xi \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p_3}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} \frac{d^3 p'_3}{(2\pi)^3} \\
&\quad \times (2\pi)^6 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \delta^3(\vec{k}'_1 + \vec{k}'_2 + \vec{k}'_3) \\
&\quad \times \psi^*_{J',\mu'}(\vec{p}'_\rho, \vec{p}'_\lambda; \mu'_1, \mu'_2, \mu'_3) \\
&\quad \times \psi_{1/2,\mu}(\vec{p}_\rho, \vec{p}_\lambda; \mu_1, \mu_2, \mu_3) \\
&\quad \times D^{*1/2}_{\lambda'_3 \mu'_3}\big[R_W\big(k'_3, B(v_{\text{out}})\big)\big] \langle p'_3, \lambda'_3 | \\
&\quad \left(-\sqrt{\frac{2\pi\alpha_E}{\omega}} J_+\right) | p_3, \lambda_3 \rangle D^{1/2}_{\lambda_3 \mu_3}\big[R_W\big(k_3, B(v_{\text{in}})\big)\big] \\
&\quad \times D^{1/2}_{\mu'_1 \mu_1}\big[R_W\big(k_1, B^{-1}(v_{\text{out}}) B(v_{\text{in}})\big)\big] \\
&\quad \times D^{1/2}_{\mu'_2 \mu_2}\big[R_W\big(k_2, B^{-1}(v_{\text{out}}) B(v_{\text{in}})\big)\big] \\
&\quad \times \delta^3\big(k'_1 - B^{-1}(v_{\text{out}}) B(v_{\text{in}}) k_1\big) \\
&\quad \times \delta^3\big(k'_2 - B^{-1}(v_{\text{out}}) B(v_{\text{in}}) k_2\big),
\end{aligned} \tag{5}$$

where $\alpha_E = \frac{1}{137}$, $\epsilon = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$, and $J_+ = -\frac{1}{\sqrt{2}}(J_x + i J_y)$. In Eq. (5) $p'_i = B(v_{\text{out}}) k'_i$, and $p_i = B(v_{\text{in}}) k_i$. μ and μ' are the projections of the angular momenta of the initial nucleon (1/2) and its final resonance (J'), respectively. The two wave functions in Eq. (5) are the intrinsic wave functions of the initial nucleon and final resonance with the momentum conjugates p_ρ and p_λ. ξ is the sign of the pionic decay of the resonance. Due to the symmetry of the wave functions, it is sufficient to consider only one case where quark 3 is struck by the incoming photon, while the other two are spectators, and to simply multiply the results by a factor of 3. The conventional transition amplitude $A_{1/2}$ or $A_{3/2}$ is determined from Eq. (5) by setting the quantum numbers of the initial nucleon (μ) and of the final resonance (μ') to be $-1/2$ and $1/2$, or $1/2$ and $3/2$, respectively.

In a similar way, one can calculate the longitudinal electromagnetic transition amplitude based on the relativistic point form. Refs. [24–26] show the calculations of the longitudinal amplitude $S_{1/2}$ in the NRCQM with the relativistic corrections. One can define the electromagnetic longitudinal transition amplitude $C_{1/2}$ in terms of the matrix element of the longitudinal electromagnetic interaction H^L_{em} [11,25]

$$\begin{aligned}
C_{1/2} &= \xi \langle f, \mu' | H^L_{\text{em}} | i, \mu \rangle = \xi \langle f, \mu' | \sqrt{\frac{2\pi\alpha_E}{\omega}} \epsilon^L_\nu \cdot J^\nu | i, \mu \rangle \\
&= 3\xi \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p_3}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} \frac{d^3 p'_3}{(2\pi)^3} \\
&\quad \times (2\pi)^6 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \delta^3(\vec{k}'_1 + \vec{k}'_2 + \vec{k}'_3) \\
&\quad \times \psi^*_{J',1/2}(\vec{p}'_\rho, \vec{p}'_\lambda; \mu'_1, \mu'_2, \mu'_3) \\
&\quad \times \psi_{1/2,1/2}(\vec{p}_\rho, \vec{p}_\lambda; \mu_1, \mu_2, \mu_3) \\
&\quad \times D^{*1/2}_{\lambda'_3 \mu'_3}\big[R_W\big(k'_3, B(v_{\text{out}})\big)\big] \langle p'_3, \lambda'_3 | \\
&\quad \sqrt{\frac{2\pi\alpha_E}{\omega}} \frac{\sqrt{Q^2}}{q} J_0 | p_3, \lambda_3 \rangle D^{1/2}_{\lambda_3 \mu_3}\big[R_W\big(k_3, B(v_{\text{in}})\big)\big] \\
&\quad \times D^{1/2}_{\mu'_1 \mu_1}\big[R_W\big(k_1, B^{-1}(v_{\text{out}}) B(v_{\text{in}})\big)\big] \\
&\quad \times D^{1/2}_{\mu'_2 \mu_2}\big[R_W\big(k_2, B^{-1}(v_{\text{out}}) B(v_{\text{in}})\big)\big] \\
&\quad \times \delta^3\big(k'_1 - B^{-1}(v_{\text{out}}) B(v_{\text{in}}) k_1\big) \\
&\quad \times \delta^3\big(k'_2 - B^{-1}(v_{\text{out}}) B(v_{\text{in}}) k_2\big),
\end{aligned} \tag{6}$$

where the photon three-momentum is selected to be $\vec{q} \parallel \hat{z}$ (with $q^\mu = (\omega, \vec{q})$ and $\vec{q} = (0, 0, q)$). This definition of the longitudinal transition amplitude $C_{1/2}$ is consistent with that of the transverse transition amplitude in Eq. (5). Usually, the polarization vector of the longitudinal polarized photon is selected to be $\epsilon^L_\mu = (\frac{q}{\sqrt{Q^2}}, 0, 0, \frac{\omega}{\sqrt{Q^2}})$, so that the constraint condition $q^\mu \epsilon^L_\mu = 0$ is satisfied [24–26]. For the longitudinal photon quark vertex, the electromagnetic interaction is, therefore, $H^L_{\text{em}} = \epsilon^L_0 J^0 - \epsilon^L_3 J^3$. The gauge invariant condition $q^\mu J_\mu = 0$ gives that $\langle f | H^L_{\text{em}} | i \rangle = \frac{\sqrt{Q^2}}{q} \langle f | J_0 | i \rangle$. Clearly, the longitudinal electromagnetic interaction in Eq. (6) is proportional to $\sqrt{Q^2}$, and it vanishes in the real photon limit $Q^2=0$. The interaction $H^L_{\rm em}$ is a Lorentz scalar. Thus, the results of the longitudinal transition amplitude $C_{1/2}$ (Eq. (6)) in the fully relativistic point-form are frame-independent too. It should be mentioned that the conventional longitudinal transition amplitude $S_{1/2}$, defined in Refs. [24–26], is

$$S_{1/2}=\xi\langle f,\mu'|\sqrt{\frac{2\pi\alpha_E}{\omega}}J_0|i,\mu\rangle. \tag{7}$$

It is not a Lorentz invariant amplitude. The two definitions of the longitudinal transition amplitudes in Eqs. (6) and (7) are different by a factor of $\frac{\sqrt{Q^2}}{q}$.

We know that the dynamically determined current (see Eq. (4)) does influence the longitudinal current in the point form because the four momenta of the system are Hamiltonians. In addition, we noted that the data for the transition amplitudes in Particle Data Group (PDG) [27] are $A^N_\lambda(X)\cdot A_{X\to\pi N}/|A_{X\to\pi N}|$. The sign $A_{X\to\pi N}/|A_{X\to\pi N}|$ of the coupling $A_{X\to\pi N}$ is involved in those amplitudes since one cannot determine it in elastic $N\pi$ scattering [28,29]. Therefore, the phases ξ have to be individually calculated in each model to avoid any confusion in comparing the model calculations with the data, especially in the critical cases as for the Roper resonance [28,29]. This means that in a theoretical calculation, both the electromagnetic and pion couplings to the nucleon have to be calculated simultaneously. Ignorance of the pion coupling part might lead to incorrect results for the sign of the transition amplitudes.

3. The electromagnetic transition amplitudes and the E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} ratios in $\gamma N\to\Delta(1232)$

In our numerical calculations, the conventional harmonic oscillator wave functions are simply employed for the nucleon and the $\Delta(1232)$ resonance. The two parameters in the calculations are selected to be 0.16 GeV^2 for harmonic oscillator constant α, and $m_q=M_N/3$ for the u and d constituent quark masses. Thus, the frequency of the harmonic oscillator is about 500 MeV which is required by the mass spectra of the nucleon resonances in some calculations of the NRCQM [30,31], where the mass gap between the nucleon and its first orbital excitations $S_{11}(1535)$ and $D_{13}(1520)$ are determined by the confining potential. Several experimental groups are now analyzing the exact values for the ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} at $Q^2=0$. The result of E_{1^+}/M_{1^+} from Mainz group is $(-2.5\pm0.2\pm0.2)\%$ [32]. Other independent analyses of the ratio are $(-3.19\pm0.24)\%$ from RPI group [33] and $(-3.0\pm0.3\pm0.2)\%$ from LEGS [34], respectively. So far, the extraction of the resonance contributions to the ratios from the experimental data is not easily performed and is still a matter of debate. However, all the present analyses agree with a large value for the ratio $|E_{1^+}/M_{1^+}|>2\%$. Recently JLAB shows the Q^2-dependence of the two ratios [35,36]. It is interesting to mention that perturbative QCD constrains the two ratios to be $E_{1^+}/M_{1^+}=1$ and $S_{1^+}/M_{1^+}=\text{const}$ when Q^2 is large in the perturbative QCD region. These constrains tie to real photoproduction data and to un-separated resonance response functions [37]. The two ratios have also been carefully analyzed by many work in isobar model (see for example Ref. [38]) and by the constituent quark models [39]. In Ref. [36], it is found that none of the soft approaches of the constituent quark models gives a satisfactory description of the data.

To proceed with a practical calculation, we know that the two ratios E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} relate to the well-known transition amplitudes. Here, the amplitudes are

$$\begin{aligned}
E_{1^+}&=-\frac{1}{2\sqrt3}(\sqrt3A_{1/2}-A_{3/2}),\\
M_{1^+}&=-\frac{1}{2\sqrt3}(3A_{3/2}+\sqrt3A_{1/2}),\\
S_{1^+}&=-\frac12S_{1/2}.
\end{aligned}\tag{8}$$

When the configuration mixing is not considered in the NRCQM, we have $A_{3/2}=\sqrt3A_{1/2}$ and $S_{1/2}=0$ and moreover, the ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} vanish for the $\Delta(1232)$ resonance. Our previous work shows [11] that although the PFCQM provides non-vanishing results for the two ratios due to the relativistic effect, the obtained values are about two orders smaller in the magnitude than the experimental data. To improve the model calculation, we notice that the D-wave deformations in the nucleon and the $\Delta(1232)$ resonance wave functions can provide an important contribution to the two ratios. When the configuration mixing effect is included, the explicit forms of the wave functions become [3,30]

$$\begin{aligned}
|\Delta\rangle&=b_S\big|\Delta^4S_{3/2}\big\rangle_S+b_{S'}\big|\Delta^4S'_{3/2}\big\rangle_S\\
&\quad+b_D\big|\Delta^4D_{3/2}\big\rangle_S+b_{D'}\big|\Delta^2D'_{3/2}\big\rangle_{M'}
\end{aligned}\tag{9}$$

for the $\Delta(1232)$ resonance, and

$$\begin{aligned}
|N\rangle&=a_S\big|N^2S_{1/2}\big\rangle_S+a_{S'}\big|N^2S'_{1/2}\big\rangle_S\\
&\quad+a_{S''}\big|N^2S_{1/2}\big\rangle_M+a_D\big|N^4D_{1/2}\big\rangle_{M'}
\end{aligned}\tag{10}$$

for nucleon. In Eqs. (9), (10) a_S, $a_{S'}$, $a_{S''}$, a_D, b_s, $b_{s'}$, b_D, and $b_{D'}$ are the configuration mixing coefficients.

In Figs. 1, 2, the calculated transition amplitudes of the $\Delta(1232)$ resonance in the PFCQM with the configuration mixing effect are plotted compared with the data and the results of the NRCQM in the Breit frame. In the figures, the configuration mixing coefficients are simply borrowed from the results of Isgur, Karl and Koniuk [30] and of Capstick [3]. Here, we stress that the coefficients of the D-wave admixture are $b_D=-0.1094$, $b_{D'}=0.0740$ for the $\Delta(1232)$ resonance and $a_D=-0.0494$ for the nucleon. It is argued, in the literature, that the small values for the D-wave probabilities (less than 2% and 0.3% in Δ and nucleon, respectively) limit the two ratios in the NRCQM [41]. The data in Figs. 1, 2 are from the compilation of Refs. [38,40]. Figs. 1, 2 show that the results of the present PFCQM and of the NRCQM are remarkably different when Q^2 larger than 0.5 GeV^2. The larger Q^2 is, the more obvious discrepancy between the two frameworks will appear. Since the amplitudes $A_{1/2,3/2}$ are multiplied by Q^3 in Figs. 1, 2,

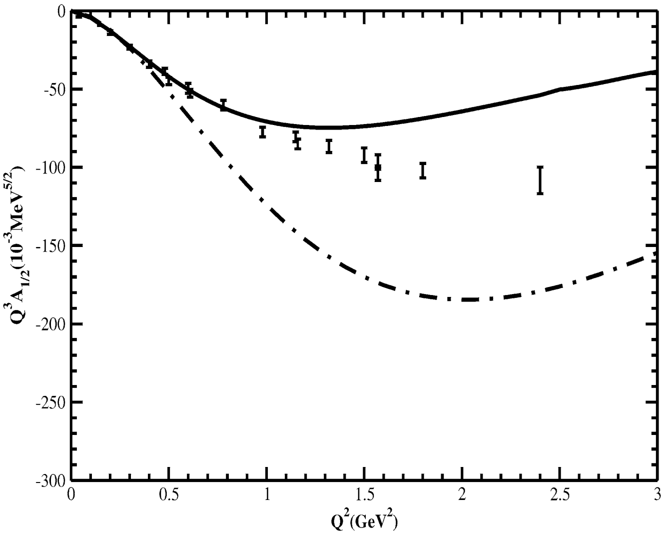

Fig. 1. $Q^3 A_{1/2}(\times 10^{-3}\ \mathrm{GeV}^{2/5})$ of the $\gamma N \to \Delta(1232)$ transition. The solid and dotted-dashed curves are the results of PFCQM and of NRCQM, respectively. The data are from the compilation of Refs. [38,40].

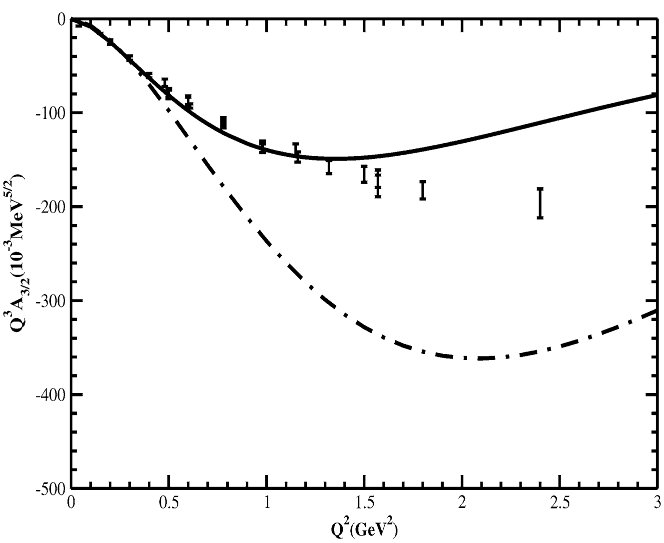

Fig. 2. $Q^3 A_{1/2}(\times 10^{-3}\ \mathrm{GeV}^{2/5})$ of the $\gamma N \to \Delta(1232)$ transition. Notations as in Fig. 1.

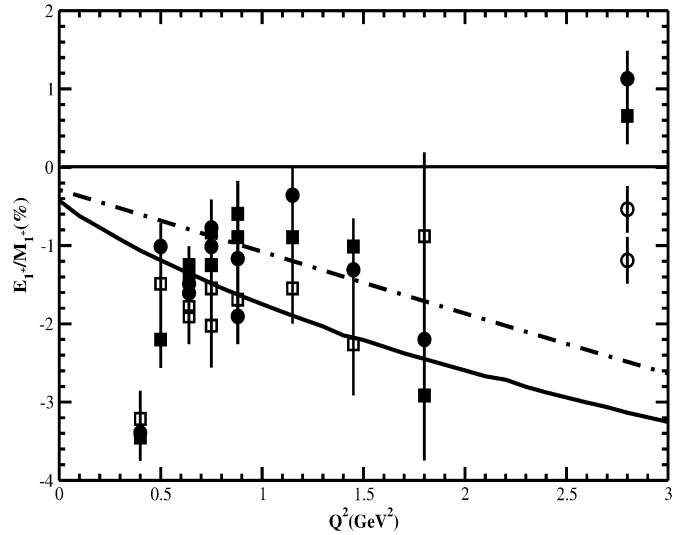

Fig. 3. Ratio of E_{1+}/M_{1+} of the $\gamma N \to \Delta(1232)$ transition. The solid and dotted-dashed curves are the results of PFCQM and of NRCQM, respectively. The data are from the compilation of Ref. [26].

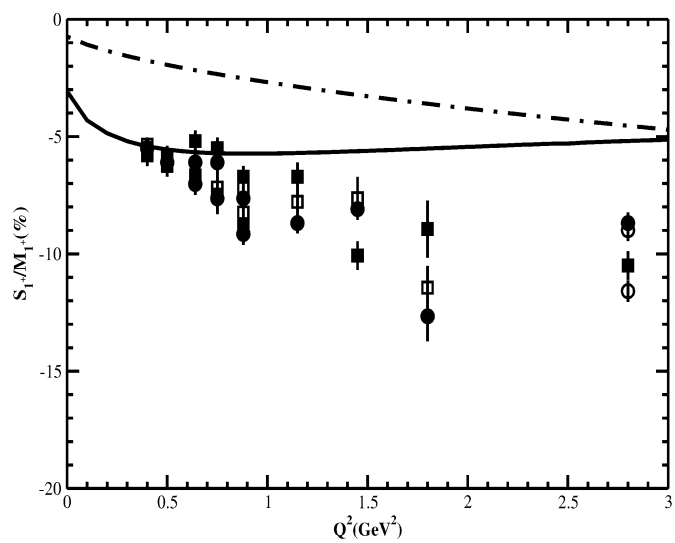

Fig. 4. Ratio of S_{1+}/M_{1+} of the $\gamma N \to \Delta(1232)$ transition. Notations as in Fig. 3.

the difference between the two frameworks are amplified when $Q^2 > 1\ \mathrm{GeV}^2$. Our calculation shows that the point-form relativistic framework is better than the NRCQM and it can well describe the experimental measurement in the region of Q^2 less than $1.5\ \mathrm{GeV}^2$.

Furthermore, we can calculate the ratios of E_{1+}/M_{1+} and S_{1+}/M_{1+}. Figs. 3, 4 show the calculated results for the two ratios in the PFCQM comparing with the data from the compilation of Ref. [36], and the values of the NRCQM in the Breit frame. Figs. 3, 4 show that the magnitudes of the ratios of the PFCQM are remarkably larger than the values of the NRCQM. One reason is due to the fact the magnitudes of transition amplitudes $A_{1/2,3/2}$ estimated in the PFCQM decrease with Q^2 faster than those of the NRCQM as seen in Figs. 1, 2. The large differences between the two frameworks indicate the important role of the relativistic effect on the Q^2-dependences of the two ratios. In the real photon limit, the magnitude of the ratio E_{1+}/M_{1+} in the PFCQM is about 50% larger than that of the NRCQM, whereas the value of S_{1+}/M_{1+} in the PFCQM is about three times larger than the one of the NRCQM because of the significant contribution of the dynamically determined current. These features are favored by the experimental measurement. We also find that the Q^2-dependence of S_{1+}/M_{1+} in the PFCQM is improved comparing with the NRCQM. Therefore, the results in Figs. 3, 4 imply that the relativistic PFCQM are more favored by the data than those of NRCQM due to the sizable relativistic effect on the ratios E_{1+}/M_{1+} and S_{1+}/M_{1+}. Our results in Figs. 3, 4 illustrate that S_{1+}/M_{1+} of the PFCQM becomes almost Q^2-independent when Q^2 is larger than about $0.8\ \mathrm{GeV}^2$, and E_{1+}/M_{1+} decrease continuously.

In Ref. [15], power-law wave functions of the nucleon and Δ are employed. The wave functions with the qualitative configuration mixing are

$$|\Delta\rangle = b'_S \big|\Delta^4 S_{3/2}\big\rangle_S + b'_D \big|\Delta^4 D_{3/2}\big\rangle_S, \tag{11}$$

for the $\Delta(1232)$ resonance and

$$|N\rangle = a'_S \big|N^2 S_{1/2}\big\rangle S + a'_D \big|N^4 D_{1/2}\big\rangle_{M'} \tag{12}$$

for the nucleon with b'_D and a'_D being ± 0.2, respectively. In that work, N–Δ transitions are studied in a schematic Poincaré covariant quark model in the three forms of the relativistic kinematics. Comparing our present calculation to the results of Ref. [15] in the point-form, one sees that our predicted values for the ratio E_{1^+}/M_{1^+} are qualitatively similar to the two cases of $(a'_D, b'_D) = (-0.2, 0)$ and $(0, -0.2)$ in Ref. [15]. However, the differences appear in the ratio S_{1^+}/M_{1^+}, since the positive values were obtained in all cases of the deformations (see Ref. [15]). This is mainly because we consider the dynamically determined current Eq. (4) which affects the results of the amplitudes S_{1^+}.

4. Conclusions

In this work, we use the point-form relativistic quantum mechanics to study the $\gamma N \to \Delta(1232)$ transition amplitudes, and particularly, the ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+}. The BJ method to implement the PF framework is employed. As has been stressed by Ref. [19], this BJ method implies that the physics is described on a hyperplane [17]. It differs from the original Dirac one in which the physics is defined on a hyperboloid surface. In our calculation, a realistic configuration mixing effect is taken into account. Moreover, we consider the dynamical determined current J^{DD} which affects the longitudinal current. Our results show that the relativistic framework can enlarge the magnitudes of the ratios of the NRCQM evidently and show the importance of the relativistic effect. Since the values of M_{1^+} and E_{1^+} are frame-independent in the point-form relativistic quantum mechanics, our results for the ratio E_{1^+}/M_{1^+} is frame-independent too. Here, we simply use the results of Refs. [3,30] for the configuration mixing. Our estimated values for the two ratios E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} imply that although the D-wave deformations of the $\Delta(1232)$ and nucleon wave functions are small (less than 2% and 0.3%), the ratio E_{1^+}/M_{1^+} (about -0.5%) in the PFCQM at the real photon limit is manifestly larger than the results of NRCQM (-0.3%). However, the two values are still much less than the measurement. It indicates that the present relativistic PFCQM still cannot reproduce the data well even though an obvious improvement has been obtained. This conclusion is not surprising, because it is expected that the inconsistencies of the two ratios remains in the three constituent quark model as pointed by Refs. [36,41]. Here, our calculations show the sizable differences between the point-form relativistic frame work and the NRCQM. The differences also illustrate that the transition amplitudes in the PFCQM are better than those in the NRCQM.

One way to remedy the inconsistency of the two ratios of E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} in the real photon point is to take the meson cloud effect into account. In fact, the pion cloud effect is believed to be one major source for the two ratios [32,42]. Here, only the contribution of the three-quark core is taken into account. The harmonic oscillator constant $\alpha = 0.16\ \mathrm{GeV}^2$ implies that the size of the three-quark core is small (about 0.5 fm). In this work, we simply use the harmonic oscillator-type wave functions for the nucleon and $\Delta(1232)$ resonance (see Ref. [30]). This treatment has its own disadvantage [43]. Furthermore, we only use the simple three-dimensional harmonic oscillator wave functions. It is argued and expected that the wave functions of four-dimensional fully relativistic harmonic oscillator [44] or of the realistic hyper-central potential model [45,46] can provide more reliable results in the relativistic calculation. Much better results for the transition amplitudes and for the two ratios E_{1^+}/M_{1^+} and S_{1^+}/M_{1^+} are expected if other physical ingredients, like the realistic wave functions and meson cloud effect, are consistently included.

Finally, the applicability of the point-form relativistic description to meson (like pion meson) properties is still a matter of debate [14,47]. It appears that the radius of the bound state wave function is an important parameter distinguishing the relativistic effect. In the calculation of pion meson electromagnetic form factors [14], one finds that space–time translation invariance is seriously broken. It means the momentum transferred to the total nucleon is different from the momentum transferred to a struck constituent. For the nucleon case, we calculate the relation

$$-C\big(Q^2\big)\langle\Delta|Q^2 J_+(x)|N\rangle = \langle\Delta|(p_3 - p'_3)^2 J_+(x)|N\rangle \tag{13}$$

and find that the factor $C(Q^2)$ is about 3–5 when $Q^2 > 0$. It should be mentioned that the space–time translation invariance is violated by a large factor (>30) in the PF calculation of the pion meson electromagnetic form factors. Thus, it is believed that the present PF calculation of the form factors of the nucleon and its excitations is more reliable than that of the pion meson. In addition, a new study of the N–Δ transition based on the PFCQM defined on a hyperboloid surface [19] is also of interest.

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